Improving the Performance of Volatility-Managed Portfolios*

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Abstract

Recent studies have criticized volatility-managed portfolios for two reasons: poor out-of-sample performance and inaccessible abnormal returns owing to transaction costs. We propose a simplified and more robust method of volatility-timing by focusing on downside forecasting using past volatility. The proposed trading strategies are free of look-ahead bias and have low trading costs. We show that downside-managed portfolios outperform unmanaged portfolios and volatility-managed portfolios. Lastly, we show that downside-managed portfolios embed a mechanism to ideally balance between type I and type II errors in downside forecasting under return asymmetries.

JEL Classification: G10, G11, G12

Keywords: Volatility-managed portfolio; downside prediction; portfolio choice

1 Introduction

A pearl of common wisdom in investment is that long-term investors should refrain from market timing and ignore short-term market volatility. Recent studies (Moreira and Muir (2017), Moreira and Muir (2019)) challenge this view and argue that investors could benefit from volatility-managed portfolios, which are constructed to have a reduced risk exposure after an increase in volatility. However, Cederburg et al. (2020) show that volatility-managed portfolios are not implementable in real-time and have worse out-of-sample performance than the original unmanaged portfolios. In another study, Barroso and Detzel (2021) find that the limit to arbitrage prevents the correction of mispricing indicated by the positive spanning alpha of volatility-managed portfolios. This evidence calls into question the value of volatility timing for real-time investors facing trading frictions and lends support to traditional wisdom. In this paper, we propose a method of volatility-timing in real-time to improve out-of-sample performance and provide new insights into the benefits of market timing portfolios for long-term investors.

As pointed out by Cederburg et al. (2020), the effectiveness of a volatility-managed portfolio depends upon the relation between lagged volatility and future expected returns, and this risk-return trade-off could be unstable over time. The most well-known example is the risk-return relation for the market portfolio, with ambiguous relations found in the literature (e.g., Glosten et al. (1993) show that both directions are consistent with theory). Volatility management suggests that investors should sell when observing high market volatility. However, the standard financial advice is that a sudden rise in volatility is typically associated with panic selling, which can be attractive buying opportunities. Furthermore, Martin (2017) shows that the expected return on the market is high when the option-implied volatility is high, and investors should buy during periods of high market volatility. These seemingly contradictory suggestions stem from different underlying assumptions about the relation between the lagged volatility and future realized returns. These assumptions are difficult to verify in real-time or out-of-sample.

We proposed a downside-managed portfolio, which simplifies the underlying assumptions and focuses directly on the core relation behind the volatility timing argument, the relation between lagged volatility shoot-up and future market downside. In our view, this is of first-order importance in volatility timing because investors are more concerned about avoiding market crashes than fine-tuning the stocks' positions based on volatility. While finding a strong risk-return relation might be elusive for the market portfolio, market volatility can provide useful information about the future market downside. The separation of downside prediction and calculating portfolio weights is essential in out-of-sample performance for two important reasons. First, it is difficult to obtain robust risk-return trade-offs, especially out-of-sample. Second, Lochstoer and Muir (2022) show that investors have slow-moving beliefs about stock market volatility, and an initial rise in volatility could signal further downturns in the stock market.

Empirically, Table 1 and Figure 1 provide intuition for our portfolio. We first check the in-sample covariance between the Sharpe ratio and lagged volatility in Table 1. Past volatility significantly correlates with the future Sharpe ratio for the market factor (consistent with Moreira and Muir (2017)), but the relation is not systematically significant among nine factors (with 4 out of 9 being significant). If we look at the relation for market factor in a real-time scenario, as shown in the top panel of Figure 1, the real-time relation is not stable and significant over time. This provides direct evidence to show why a volatility-managed portfolio does not work in real time. We also find that the covariance between return and lagged volatility is insignificant and unstable over time. In contrast, past volatility can persistently and significantly predict future Value-at-Risk (VaR), associated with a higher probability of large negative returns. Motivated by this observation, we construct downside-managed portfolios by dynamically switching between buy-and-hold and reduced stock positions based on predicting the normal or downside states.

The goal of downside management is to avoid extreme negative returns. Our approach relies on the insights from the literature on probability density forecasts, which have become a standard practice in many areas of economics and finance (see, e.g., Diebold et al. (1998), Granger and Pesaran (2000), Corradi and Swanson (2006)). Applications include optimal business cycle turning point forecasts (Zellner et al. (1991)), and financial risk management using certain distribution aspects, such as value-at-risk (VaR). Econometric models have some success in forecasting conditional densities or higher-order moments even though they have difficulty forecasting conditional means (see, e.g., Diebold et al. (1998) and Hong et al. (2007)).

Our approach to downside prediction consists of downside probability estimation and parameter optimization. We estimate downside probability by logistic regression with rolling training periods, and search the optimal model parameter by maximizing the F_β score. The F_β score is a useful measure for prediction accuracy in situations where the outcome is unbalanced, such as downside events, which are unlikely to occur relative to the alternative. By construction, the F_β score penalizes type I errors (false positives) and type II errors (false negatives). β is the parameter measuring the relative weights of two types of errors. We use the F_β score for parameter optimization rather than other portfolio performance measures such as the Sharpe ratio because the F_β score is a direct measure of downside prediction accuracy, and because the relationship between volatility and downside is more stable. When the relationship between volatility and performance measure is unstable, the parameter choice from in-sample optimization cannot guarantee superior out-of-sample performance.

With our approach, we first find that the real-time performance of the downside-managed portfolio is significantly better than the original unmanaged portfolio. Specifically, we evaluate the performance using different metrics, including Sharpe ratio(SR), Certainty equivalent return(CER), Certainty equivalent return with asymmetric preference(CER^{asy}), and maximum drawdown. The real-time downside-managed portfolios of nine major factors exceed the original factors in all cases measured by Sharpe ratio (with seven significant differences) and beat in 7 cases measured by CER (with five significant differences). In a broad sample, downside-managed portfolios outperform 63 anomaly portfolios (32 significant beats) out of 94 anomalies (summarised by Hou et al. (2015) and McLean and Pontiff (2016)) measured by the Sharpe ratio and beat 88 portfolios (66 significant beats) measured by CER. We also show that the downside-managed portfolios achieve lower maximum drawdown and enhance the utility gains of investors with asymmetric preferences.

Second, we explore the option-implied volatility about future market returns and find options containing important information to better predict future market downturns. The managed market portfolio using the options data achieves a much higher Sharpe ratio than the original market portfolio and the managed portfolio with lagged volatility.

Third, we test the performance of the managed portfolios after transaction costs. The results are robust by assuming different levels of monthly trading costs and by comparing the break-even transaction costs. The downside-managed portfolios have much lower turnover ratios than the volatility-managed portfolios and, therefore can be better applied in a setting with high transaction costs.

Volatility timing might not work all the time and for all return distributions. Why would our strategy perform better for some portfolios than others in our setting? The answer to this question could highlight the mechanism of our strategy and the appropriate situations to implement it. Conceptually, volatility-timing allows investors to deviate from the return distributions of the original unmanaged portfolios. Theoretically, investor preference and model prediction accuracy jointly determine the appropriate degree of deviation in distribution through volatility timing. Many studies have highlighted the implications of investor preference on portfolio choice (Gul (1991); Routledge and Zin (2010); Polkovnichenko et al. (2019)). Our approach allows the separation of these two determinants and singles out the role of model prediction accuracy. In our approach, maximizing the F_β score for a given β matters for prediction accuracy. β is an investor's choice in switching between the risk-free asset (in the predicted downside state) and the original unmanaged portfolio (in the predicted normal state). Specifically, we show that β choice depends on the prediction model and the return asymmetry of unmanaged portfolios. More aggressive downside management is appropriate for better prediction models and more negatively skewed return distribution.

Our study contributes to the literature on the value of volatility timing. We provide an improved volatility timing method that is robust to trading costs and estimation risk and show that it can benefit mean-variance investors and mean-variance-asymmetry investors in real-time implementation.

Our study adds to the understanding of portfolio choice theory. Within our approach, the traditional buy-and-hold portfolios are optimal for investors who are extremely averse to false positives in downside prediction. Conversely, risk-free assets are optimal for investors extremely averse to false negatives in downside prediction. Missing out on the upside from false positives and suffering from the downside from false negatives is an inevitable trade-off.

Our study also relates to the recent literature on "p-hacking". Our approach separates model prediction accuracy and investor preference. Harvey (2017) raises the concern that the researchers yield an embarrassing number of false positives in anomaly discoveries due to multiple testing and selection bias. Since researchers could have different attitudes toward false positives from investors, sub-optimal predictive models may have been deployed in anomaly discoveries. Our approach directly deals with false positives by simplifying volatility-timing to downside prediction. This is an alternative approach to more restricted criteria introduced to reduce the number of false discoveries (Benjamini and Hochberg (1995); Benjamini and Yekutieli (2001); Harvey and Liu (2014); Harvey (2017)).

The rest of the paper is organized as follows. Section 2 describes our methodology and model implications, followed by the data and sample in section 3. We evaluate the out-of-sample performance of the strategies in Section 4. Section 5 provides some further analysis of the strategy. Finally, we conclude in Section 6.

2 Methodology

In this section, we introduce our methodology to construct the downside-managed portfolios. We first describe the model to predict the downside state. Then we illustrate how we adjust our positions of portfolios according to the predicted state. We also discuss the economic mechanism of the parameter level in our model and its role in optimal portfolio choice. Consider an asset with return series r_t, where $t = 1, 2, 3, ..., T$. We assume that over time its price switches between a normal state and a downside state, under which it experiences a crash. At the beginning of each time period t (or the end of the previous time period $t-1$), we predict the state at time t, and determine the position of our downside-managed portfolios. If we predict a normal state, then we take a full position on the asset, i.e., we hold the asset. Otherwise, if we predict a downside state, then we reduce our position accordingly. Therefore, the predictive model is key to our downside-managed portfolios.

We adopt the classification methodology, one of the supervised learning techniques, to conduct predictive analytics with categorical outcomes. We first use logistic regression with lagged volatility to estimate the downside probability, then we search for the optimal threshold maximizing predictive performance. Finally, the position for the coming time period t depends on the predicted state by comparing the estimated probability and the optimal threshold. The method combines logistic regression and binary classification, a fundamental practice in machine learning. Our goal is to find a sensible model with high predictive power and to illustrate the potential of downside management via volatility-timing. Moreover, our method could be adapted to different preferences and different return distributions of the managed portfolio.[1]

[1]Our downside-managed portfolios are not limited to the predictive model described in this subsection. In this paper, we adopt a classic model to examine the performance of the downside-managed portfolios. As more accurate prediction a better construction of the portfolios, applying a more advanced predictive model could potentially increase the performance of the portfolios.

To predict the state at time t, we consider the following logistic regression:

$$log\frac{Pr(Down_\tau)}{1 - Pr(Down_\tau)} = \delta_0 + \delta_1\sigma_{\tau-1}, \tag{1}$$

where $Pr(Down_\tau)$ is the probability of the downside state at time τ, $\sigma_{\tau-1}$ is the volatility estimated at time $\tau - 1$, and $\tau = 2, ..., t-1$.

We consider two ways of estimating the return volatility σ_τ. First, we estimate it based on returns with higher frequency. For example, when we consider monthly trading strategies, we follow Moreira and Muir (2017) and use daily returns during month τ to estimate σ_τ:

$$\sigma_\tau = \sqrt{\sum_{j=1}^{N_\tau} f_{j,\tau}^2}, \tag{2}$$

where $f_{j,\tau}$ represents the return on the j^{th} trading day in month τ, and N_τ is the number of trading days in month τ.

We also consider the SVIX index introduced in Martin (2017) as an alternative measure of σ_τ. The SVIX index is a measure that extracted from option prices that capture the future risk-neutral variance of the return of the underlying asset, which also contains useful information in predicting the future return. We discuss the estimation of the SVIX index in Section 3.

For each time $\tau = 2, 3, ..., t-1$, we define a crash dummy $Down_\tau$ as a binary dependent variable:

$$Down_\tau = \begin{cases} 1 & \text{if} \quad r_\tau < r_{\tau-1}^{var}; \\ 0 & \text{if} \quad r_\tau > r_{\tau-1}^{var}, \end{cases} \tag{3}$$

where r^{var} is a pre-specified percentile of the empirical return distribution from time 1 to time $\tau - 1$.

With return volatilities, $\sigma_1, \sigma_2, ..., \sigma_{t-2}$, and crash dummies, $Down_2, Down_3, ..., Down_{t-1}$, we fit model (1) and obtain parameter estimation $\hat{\delta}_{0,t}$ and $\hat{\delta}_{1,t}$. Together with return volatility of time $t-1$, σ_{t-1}, we get fitted log odds, $\hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{t-1}$, and use it to predict the state at time t:

$$\widehat{Down}_t = \begin{cases} 1 & \text{if} \quad \hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{t-1} > y_t; \\ 0 & \text{if} \quad \hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{t-1} \le y_t, \end{cases}$$

where y_t is some threshold chosen based on information up to time $t-1$. When $\widehat{Down}_t = 1$, we predict a downside state for time t, and a normal state otherwise.

We next illustrate our choice of optimal y_t. With estimates of $\hat{\delta}_{0,t}$ and $\hat{\delta}_{1,t}$, we first have a time-series of fitted log odds: $\hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{\tau}$, where $\tau = 1, 3, ..., t-2$. Any value of y_t yields a time-series of fitted state variables: $\widehat{Down}_{\tau}^{y_t}$, where

$$\widehat{Down}_{\tau}^{y_t} = \begin{cases} 1 & \text{if} \quad \hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{\tau-1} > y_t; \\ 0 & \text{if} \quad \hat{\delta}_{0,t}+\hat{\delta}_{1,t}\sigma_{\tau-1} \le y_t, \end{cases} \tag{4}$$

and $\tau = 2, 3, ...t-1$. By comparing the realized states $Down_{\tau}$ with the fitted values $\widehat{Down}_{\tau}^{y_t}$, we can classify the observations from $t = 2, 3, ..., t-1$ into four scenarios: true positive (TP), false positive (FP), true negative (TN), and false negative (FN), defined in the following table:

	$Down_{\tau} = 1$	$Down_{\tau} = 0$
$\widehat{Down}_{\tau}^{y_t} = 1$	TP	FP
$\widehat{Down}_{\tau}^{y_t} = 0$	TN	FN

We select the threshold y_t to maximize the F_{β} measure:

$$F_{\beta} = \frac{(1+\beta^2)\#TP}{(1+\beta^2)\#TP + \beta^2\#FN + \#FP}. \tag{5}$$

The choice of β reflects the balance between FP (type I error) and FN (type II error).

When $\beta = 1$, the importance between FP and FN are equal. This case corresponds to the F_1 measure that is commonly used for prediction evaluations:

$$F_1 = \frac{2\#TP}{2\#TP + \#FN + \#FP}.$$

Alternative levels of β also have important implications, which will be discussed in the next subsection.

With the empirically selected threshold y_t, we are able to give the prediction of state at time t by (4), and determine the position of our managed portfolilo at time t:

$\widehat{Down}_t$	Predicted state	Position
1	Downside	0.5
0	Normal	1

We then repeat the procedure for all time periods t in our sample to construct the managed portfolios. It is important to notice that with this procedure, the constructed strategy does not suffer from look-ahead bias. This is because determining the position for time t does not include any future information.

The downside managed portfolios have several appealing features. First, the weight of the asset depends on a two-state prediction, such that the corresponding position does not have to change frequently. Such infrequent rebalancing differs from the volatility-managed portfolios that typically require the investor to conduct monthly rebalancing based on past volatility. Second, the position of the portfolio is always positive and smaller than one, indicating that the downside portfolio does not have to take leveraged position or short-selling position. If we consider a factor portfolio as the baseline asset, it is typically a long-short portfolio by construction. Flipping sign on weight can cause the switching position from buying to selling, which induces large transaction costs. Lastly, to highlight the effectiveness of our model, it is important to note that our model is based on downside forecasting instead of mean forecasting.

2.1 Beta and portfolio choice

As shown in equation (5), the F_β measure decreases in the count of FP and FN, corresponding to type I and type II errors. When we predict future state with the logistic model described above, we face the inevitable trade-offs between the two types of errors, as one cannot reduce type I and type II errors simultaneously. Different levels of β adjust the relative weight assigned to the two types of errors. When searching for optimal threshold y_t, β adjusts the importance of avoiding type I error relative to avoiding type II error. Two types of errors are equally weighted if $\beta = 1$.[2] The tolerance to FP is higher if $\beta > 1$, and FP will be punished more relative to FN if $\beta < 1$. The tolerance to FP relative to FN is determined by investors' attitudes toward losses caused by FP compared to FN.

Economically, the two types of errors in downside prediction can have different impacts on the investor's objective function. In our setting, a higher level of β implies a "conservative" strategy by adopting a relatively lower threshold. This yields more positive predictions (predicting a downside state), and reducing the number of FN at the sacrifice of an increasing number of FP. If β is high enough, such that the model always makes downside predictions, then our managed portfolios always take a reduced position of the underlying asset. On the contrary, the other direction yields opposite results: lower β indicates an "aggressive" strategy. When β approaches zero, the managed portfolio is merely a buy-and-hold strategy. Empirically, we consider both the rule-of-thumb value of $\beta = 1$, i.e., the F_1 measure, as well as a selected value of β between the two extremes. We also discuss some empirical guidance on how to select an optimal β ex-ante in Section 5.2.

[2] The F_1 measure is commonly used for prediction evaluation.

3 Data and Sample

We collect daily and monthly data on factor excess returns for the market (MKT), size (SMB), and value (HML) factors of the Fama and French (1993) three-factor model, the momentum factor (MOM), profitability (RMW), and investment (CMA) factors of the Fama and French (1993) five-factor model, the profitability (ROE) and investment (IA) factors of the Hou et al. (2015) q-factor model, and the betting-against-beta factor (BAB) in Frazzini and Pedersen (2014).[3] The sample period starts in August 1926 for MKT, SMB, and HML; January 1927 for MOM; August 1963 for RMV and CMA; February 1967 for ROE and IA; and February 1931 for BAB. The sample periods end in December 2016.

We also collect the 94 anomaly portfolios listed in Cederburg et al. (2020), including the anomaly variables reported in Hou et al. (2015) and McLean and Pontiff (2016). We match the list of the anomalies with Cederburg et al. (2020) to directly compare the performance of our strategies with that of volatility-managed portfolios in real-time, and show the improvement in our downside-managed portfolios.

In our empirical analysis, we construct monthly re-balanced managed portfolios, i.e., we consider a month as a time period t in Section 2. Thus, we use monthly observations to calculate and evaluate portfolio performances and use daily observations to calculate return volatilities following (2). As described, we apply an expanding window to predict the state for each month t. We also require that $t > 120$ to specify an initial 120 months as the training period. Thus the out-of-sample period runs from the 121th month to the end of the sample period. The cutoff that defines a downside state in (3), r^{var}, is selected as the 5th percentile of past returns.

Besides using the past realized volatility as a predictor for downside state, we also construct alternative predictors using information extracted from option prices. We obtain S&P 500 index options data from OptionMetrics with a sample period spanning from January

[3]The data are obtained from authors' web pages.

1996 to December 2020. We consider the SVIX index constructed by Martin (2017) for market downside forecasting. Martin (2017) develops a measure for the market expected return based on risk-neutral variance implied by the prices of options written on the S&P 500 index:

$$\mathrm{SVIX}^2_{t\to T} = \frac{2}{(T-t)R_{f,t}S_t^2}\left[\int_0^{F_{t,T}} \mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \mathrm{call}_{t,T}(K)dK\right], \tag{6}$$

where $\mathrm{put}_{t,T}(K)$ and $\mathrm{call}_{t,T}(K)$ are the time-t prices of put and call options with strike price K and that mature at time T, $F_{t,T}$ is the time-t price of S&P 500 futures that mature at time T, S_t is the level of the S&P 500 index at time t, and $R_{f,t}$ is the risk-free rate. SVIX index is calculated at time t isthe annualized risk-neutral variance of the market excess return from t to T. In our empirical analysis, we calculate SVIX with options with maturity of 9-month.

Given that the variation in market downside expectation may be better captured in put option prices, we also consider two parts of the SVIX index: upside SVIX and downside SVIX. We calculate them from call and put options separately.

$$\text{SVIX-Up}^2_{t\to T} = \frac{2}{(T-t)R_{f,t}S_t^2}\int_{F_{t,T}}^{\infty} call_{t,T}(K)dK, \tag{7}$$

$$\text{SVIX-Down}^2_{t\to T} = \frac{2}{(T-t)R_{f,t}S_t^2}\int_0^{F_{t,T}} put_{t,T}(K)dK, \tag{8}$$

4 Performance of Volatility-Timing Strategies

This section tests the real-time performance of downside-managed portfolios against the original unmanaged portfolios and the volatility-managed portfolios. We first introduce various performance measures that we use to evaluate all trading strategies. Then we discuss the construction of real-time volatility-managed portfolios. Finally, we examine the real-time performance on nine major factors and 94 anomaly portfolios.

4.1 Performance evaluation measures

To evaluate portfolio performance, we use four traditional performance measures that has been widely applied in the literature, based on the mean, standard deviation, and skewness of realized returns.

First are the Sharpe ratio (SR) and the certainty-equivalent return (CER) of each portfolio:

$$SR = \frac{\hat{\mu}}{\hat{\sigma}},$$

and

$$CER = \hat{\mu} - \frac{\gamma}{2}\hat{\sigma^2},$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the sample mean and volatility of the realized returns of each strategy. For the CER, we choose the risk-aversion level of γ to be 6, but other values of γ does not change any of our conclusions. Higher CER suggests improved utility gain. To statistically distinguish the performance of two strategies, we follow Jobson and Korkie (1981) to calculate the p-value of the SR difference and DeMiguel et al. (2009) to compute the p-value of the CER difference.

The two measures above are crucial for the mean-variance investors. However, when the return distribution is skewed, and investors have a preference for the return asymmetry, mean-variance rules may no longer be as efficient (see Pedersen and Satchell (2002) and Jarrow and Zhao (2006)). To account for the return distribution asymmetry when measuring the utility gain, we adopt an alternative certainty equivalent return with asymmetry, CER^{asy}, implied by Dahlquist et al. (2017):

$$CER^{asy} = \hat{\mu} - \frac{\tilde{\gamma} - 1}{2}\hat{\sigma^2} + \tilde{\chi}\hat{\sigma}\hat{\delta},$$

where $\hat{\delta}$ is a measure that accounts for the skewness of the distribution of the realized returns, in particular, $skewness = 2\delta^3$, $\tilde{\gamma} = 6.2$, and $\tilde{\chi} = 0.0488$ according to the calibration result[4].

Finally, we also consider maximum drawdown (MDD). This measure is particularly helpful for measuring the ability to shrink the downside risk (Liu et al. (2019), Van Hemert et al. (2020)). The measure is the largest peak-to-trough cumulative return over the life of an investment. Quantitatively, we first calculate the drawdown for each year y in our sample:

$$DD_y = \frac{TV_{y-1} - PV_{y-1}}{TV_{y-1}},$$

where PV_{y-1} and TV_{y-1} are the peak value and trough value of the cumulative returns over year $y-1$. Then the MDD is calculated as:

$$MDD = \max_{y} \; DD_y.$$

4.2 Real-time volatility-managed portfolios

To fairly evaluate the performance of volatility-managed portfolios, following Liu et al. (2019), we construct the real-time version of the volatility-managed portfolios in Moreira and Muir (2017). Review that for a given asset or anomaly portfolio, a volatility-managed portfolio is constructed as:

$$f_{\sigma,t} = \frac{c}{\sigma_{t-1}} f_t,$$

where f_t is the buy-and-hold excess portfolio return in month t, and σ_{t-1} is the volatility in the previous month estimated by (2), and c is chosen as the scaling parameter, such that f_t and $f_{\sigma,t}$ have the same unconditional volatility. Volatility-managed portfolios are criticized by the in-sample estimation of c in Liu et al. (2019) since the parameter is only known to investors ex-post.

[4]According to the calibration results in Dahlquist et al. (2017), with $\tilde{\gamma} = 6.2$ and $\tilde{\chi} = 0.0488$, the asset allocation is closest to 30/70 principal

Following Liu et al. (2019) and Cederburg et al. (2020), we construct a real-time version of the volatility-managed portfolios by estimating an out-of-sample scaling parameter c relying on information up to month $t-1$:

$$f_{\sigma,t} = \frac{c_{t-1}}{\sigma_{t-1}} f_t, \tag{9}$$

wherec c_{t-1} is the real-time scaling parameter, such that

$$\sigma(f_{\sigma,\tau}) = \sigma(f_\tau),$$

where $\tau = 1, 2, 3, ..., t-1$. To keep consistent with the downside-managed portfolios, we consider the initial 120 months as the training period for the estimation of c_t and expand the window until the end of the sample period. For implementation consideration, we also impose a constraint that the leverage to be no greater than five on the weight of the volatility-managed portfolios.

4.3 Out-of-sample performance

In this subsection, we compare the out-of-sample performance of our downside-managed portfolios with the real-time volatility-managed portfolios based on the evaluation measures described in Section 4.1. We construct our downside-managed portfolio based on the F_1 measure. For illustrative purpose, we also consider a F_{β^*} measure, in which β^* is chosen to maximize the corresponding in-sample performance. (***which one? explain.***) This is the highest performance one could potentially achieve by adjusting his/her attitude between type I and type II errors of predicting a downside state. We also report the performance of the original factor as benchmarks.

4.3.1 Volatility-timing for the market

Panel A of Table 2 reports performance of different portfolios based on the S&P 500 excess returns during 1962 to 2020, with volatility estimated based on daily returns over the previous month following (2). We can see that the F_1 downside-managed portfolio outperforms the original unmanaged portfolio with higher SR (0.342 vs. 0.246), higher CER (0.006 vs. -3.232), and higher CER^{asy} (-1.094 vs. -5.061). Statistical test results show that both the SR and CER are significantly higher. In particular, the p-value for testing the difference in SR is 0.010 and that for testing the difference in CER is 0.000. Also, the F_1 downside-managed portfolio has a much less MDD of -0.304, indicating that our strategy manages downside returns well.

Our trading strategy also outperforms the volatility-managed portfolio in all metrics. For example, the CSR of our downside-managed portfolio, 0.006, is much higher than the CER of the volatility-managed portfolio, -2.003. However, consistent with Moreira and Muir (2017), for the volatility-managed portfolio, the SR and CER are higher than the benchmark, though the differences are not significant (with p-value equal 0.198 and 0.160, respectively).

It is expected that the F_{β^*} downside-managed portfolio exhibits the best performance. Its SR, CER, CER^{asy}, and MDD are 0.356, 0.582, 3.814, and -0.271, respectively. This shows the high potential that our downside-managed portfolios could obtain by adjusting investors' preference among type I and type II errors. However, it is important to notice that the optimal β^* is selected in sample for this analysis. In Section 5.2, we will further discuss some guidance that we could use in practice to select an optimal β ex-ante.

Panel B of Table 2 reports the performance of the downside-managed portfolios with volatility estimated from option prices, including SVIX, SVIX-Up, and SVIX-Down in (6), (7), and (8), respectively. Due to the availability of option pricing data, the sample period for all portfolios in Panel B spans from 1996 to 2020. Therefore, for comparison, we also include the performance measures for the four portfolios reported in Panel A over the same sample period. The downside-managed portfolios constructed with SVIX-related measures achieve much better performance than those based on realized volatility. This is what we expect, because information embedded in the option prices is forward-looking. Impressively, the downside-managed portfolio based on SVIX-Down, calculated from put options prices, has the highest SR of 0.480, the highest CER of 1.178, the highest CER^{asy} of -0.041xx, and the lowest maximum drawdown of -0.271. This is consistent with empirical findings in the literature that the put option prices could better reflect investors' expectations about the left tails of the underlying distributions, such that SVIX-Down, constructed from put option prices, better predicts the downside returns, yielding higher performance by correctly avoid the downside of the market.

4.3.2 Volatility-timing for nine major factors

Next, we study the performance of volatility-timing strategies among the nine classical asset pricing factors, including MKT, SMB, HML, MOM, RMW, CMA, ROE, IA, and BAB. Table 3 reports performance metrics of nine original factors, the corresponding volatility-managed portfolios, and downside-managed portfolios, and Table 4 presents pairwise comparison based on the difference in SR and CER. Compared with the original factors, our downside-managed portfolios have higher SR for all nine factors, with six out of nine significantly different at the 10% level. Our downside-managed portfolios also improve mean-variance utility, measured by the CER increase, in seven cases, with five significant differences. The CER^{asy} and MDD measures also exhibit substantial improvements of our downside-managed portfolios over the original factors, showing the ability of our strategy in controlling for the loss.

In contrast, the volatility-managed portfolios outperform the benchmark in five out of nine cases measured by SR, with only three significant differences (MOM, ROE, and BAB), and only three cases measured by CER, with 2 significant improvements (ROE and BAB). The downside-managed portfolios also perform well relative to volatility-managed portfolios: six higher SR with four being significant, and five higher CER with all being significant.

The downside-managed portfolios especially have appealing performance in the MMD and CER^{asy}. Compared with volatility-managed portfolios, downside-managed portfolios are better at avoiding large negative returns. A higher CER^{asy} indicates that downside-managed portfolios generate more positively skewed return distributions than the volatility-managed or unmanaged portfolios.

The volatility-managed portfolios of MOM, ROE, and BAB perform the best in this analysis, even though our downside-managed versions are not far behind. Cederburg et al. (2020) point out that the effectiveness of volatility-managed portfolios could enhance the SR when there is negative and persistent relation between lagged volatility and forward returns, as shown by the covariance term in Table 1. However, the impressive performance of volatility-managed portfolios for these three factors cannot dominate downside-managed portfolios if we allow the variation in β and predict the downside state with the F_{β^*} measure. In Panel C of Table 3, we report the performance of downside-managed portfolios using the F_{β^*} measure. We find that the downside-managed portfolios indeed achieve the best performance using the F_{β^*} measure.

4.3.3 Volatility-timing for 94 anomalies

We next compare the performance for all 94 anomaly portfolios as in Cederburg et al. (2020). For each anomaly portfolio, we make a pair-wise comparison of the SR, CER, and CER^{asy} among the corresponding downside-managed portfolio, volatility-managed portfolio, and unmanaged portfolio. Table 5 reports the number of cases where the differences in the performance measures are positive or negative. Numbers in brackets represent the number of significant positive or negative at the 5% confidence level.

Downside-managed portfolios improve the SR in 66 out of 94 anomalies compared to the original factors, with a winning rate of 70.21%. Among them, there are 32 significant increase at the 5% level. Our downside-managed portfolio also improves the SR for the volatility-managed portfolios in 45 anomalies with 22 significant increase. Considering the CER measure, downside-managed portfolios achieve even better performance. Particularly, downside-managed portfolios have a higher CER for 88 out of 94 anomalies, with 66 being significant, compared to the original unmanaged portfolios. The CER of downside-managed portfolios are also higher than that of volatility-managed portfolios for 80 anomalies, with 54 being significant. Results show that our downside-managed portfolios benefit a mean-variance investor in a very robust way. If we further consider the asymmetry of the return distribution, we see that with respect to the CER^{asy}, downside-managed portfolios still have well-matched performance: they beat the original in 81 anomalies and outperform the volatility-managed combination portfolios in 67 cases. This broad test of the portfolio performance comparison provides strong evidence that downside-managed portfolios dominate in the horse race.

5 Further Analysis

In this section, we present some further analysis to complement the empirical analysis of our downside-managed portfolios. First, we evaluate the influence of transaction costs to the performance of volatility-timing strategies. We also examine the choice of optimal β in the F_β measure, and provide some implications on how to select β ex-ante.

5.1 Transaction costs

As pointed out in Barroso and Detzel (2021), a significant amount of trading is required to implement volatility-managed portfolios. Therefore, the robustness of portfolio performance to transaction costs is an important issue in our study. Trading costs are estimated based on several reasonable assumptions. Specifically, we test a wide range of transaction costs from 1 bp to 50 bps following Wang and Yan (2021), considering the different levels of trading techniques and various liquidity demands in the real world.[5]

Table 6 provides an overview of the robustness of the volatility-timing strategies for the market by showing the SR under different levels of transaction costs. Consider volatility-managed portfolios first. With an increased transaction cost from 1bp to 50 bps, the SR of vol-managed portfolios decreases sharply, from 0.288 to 0.110 for the sample period starting from 1962, and decreases from 0.274 to 0.097 for the sample period since 1996. On the contrary, the downside-managed portfolios suffer less from transaction costs. For the sample period from 1962, the SR of the downside-managed portfolios slightly declines from 0.341 to 0.303. For the sample period from 1996, the SR almost remains quite stable for all downside-managed portfolios. For example, the SR of the downside-managed portfolio based on SVIX changes from 0.385 to 0.372 when the transaction costs increases from 1bp to 50 bps.

These results show that the performance of our downside-managed portfolio is much more robust against transaction costs than volatility-managed portfolios. This is due to the simple construction of our strategy. Instead of continuously adjusting positions according to the volatility levels as in volatility-managed portfolios, we only switch positions from full to reduced levels when we predict a downside state. Therefore, our strategy does not suffer as much from large transaction costs as volatility-managed portfolios do.

In addition to different assumptions on the trading costs, we also compute the break-even transaction costs that render the performance measures indifferent to the original unmanaged portfolios. This measure resembles the performance fee suggested in Fleming et al. (2001), which is the maximum fee an investor would be willing to pay to switch from the benchmark unmanaged portfolios to volatility-timing strategies according to a certain performance evaluation measure. A higher break-even transaction cost means that the portfolio has a higher tolerance to transaction costs while maintaining a positive difference in the performance metric relative to benchmarks.

[5]See related studies discussing trading costs differentiation: Fleming et al. (2003), Hasbrouck (2009), Asness et al. (2015), Moreira and Muir (2017), Barroso and Detzel (2021).

Table 7 and Table 8 present break-even trading costs for S&P 500 index and 9 major factors with respect to SR, CER, and CER^{asy}. Downside-managed portfolios have much higher break-even transaction costs over volatility-managed portfolios in general. For the S&P 500 index over the sample starting from 1962, the break-even transaction fee with respect to SR, CER, and CER^{asy} for the volatility-managed portfolios are \$12.67, \$21.67. and \$20.27, respectively. While for the same sample period, the corresponding break-even transaction costs are \$123.76, \$299.78. and \$410.92 for our downside-managed portfolios. Other sample period and factors give the same conclusions. Additionally, Table 9 reports the break-even transaction costs on 94 anamalies, providing further evidence of the advantage of downside-managed portfolios regarding transaction costs.

To better understand the reason why our downside-managed portfolios are more robust to transaction costs, in Table 7 and Table 8, we also report the turnover ratio for each of the portfolios of interest.[6] The turnover ratio differences between the two managed portfolios are quite large. The volatility-managed portfolios systematically require a much higher monthly turnover ratio (e.g. 81.6% for MOM and 64.7% for MKT). Such turnover ratio is extremely high even after we apply the constraint of 5 on the leverage. While the turnover ratios for downside-managed portfolios range from 5% to 6% per month. Such low turnover ratio results in that downside-managed portfolios have a higher tolerance to transaction costs, less likely to suffer from liquidity events and financial crises.

5.2 Economics of Downside Management

The relative performance of downside management with volatility timing varies across factors when compared to the unmanaged or volatility-managed portfolios. Next, we explore the potential explanations for these variations, which could also provide guidance on appropriate applications of downside management. As common with any market-timing strategy,

[6] We calculate the turnover ratios following Hasbrouck (2009).

downside management might not be optimal in every scenario.

5.2.1 Downside Management and Skewness

The performance of our downside-managed portfolios with the F_1 measure demonstrates the effectiveness of the strategy, while results based on the F_{β^*} measure show the optimal implementation of the strategy getting depends on the return distribution of the unmanaged portfolio. To illustrate this point, we examine the behavior of the strategy under different levels of β, and show both empirically and theoretically that the optimal choice of β can be related to the skewness of the original factor.

In Section 2.1, we discuss the association between the choice of β and the balance between the type I and type II errors of predicting a downside state. Now we further illustrate this relation empirically. The top-left panel of Figure 2 plots the type I v.s. type II errors of downside state prediction with different levels of β in F_β measure for each factor. A larger marker size corresponds to a higher level of β. We have several observations from the plot. First, for all of the factors, the relation between the two types of errors shows the trade-off: reducing type I error is at the sacrifice of enlarging type II error, and vice versa. Second, the performance lines are analogous to the indifference curve. If we consider that investors are constrained by the information budget, and different investors have different weights on type I and type II errors, then the optimal line is the tangent point of the information budget line with these curves.[7] Third and more importantly, we observe significant differences in the trade-off among the factors, and some factors, e.g. ROE, MOM, and RMW, see higher penalty from false negatives than from false positives in downside prediction, and thus these factors could benefit more from volatility-timing strategies.

[7]Intuitively, if the agent has zero tolerance to type I error, it is better off to hold cash. Conversely, if the agent cannot live with any opportunity cost of missing the upside potential of the asset, then the optimal choice would be the full position on the original portfolio. An interesting implication from this result is that: either buy-and-hold or full cash position is a corner solution.

To better see the investment implications of assigning different values to β, Figure 3 plots the return density of the two tails of downside-managed portfolios constructed by F_β measure with different levels of β. We show return density by defining expected tail loss (expected tail profit) as the mean of all returns that are lower (higher) than the 5th (95th) percentile of the downside-managed portfolio returns. From the relation between β and the expected tail loss/profit, we can see a clear trade-off between downside loss and upside opportunity cost. An increased value of β yields more positive predictions of the downside state, thus reducing both tails at the same time.

As β captures the degree of downside management, we conjecture that the optimal value of β is associated with the skewness of the return distribution of unmanaged portfolio. Because skewness is a measure of the asymmetry of the return distribution, a higher degree of downside management is appropriate for heavier left tails. In this section, we examine the relation between the skewness of unmanaged return distribution and the optimal β maximizing the CER^{asy}, the performance evaluation measure that accounts for return asymmetry.[8]

In this analysis, we adopt a broader sample and examine the 187 anomaly portfolios, summarized by Hou, Xue, and Zhang (2020). For each anomaly, we search the optimal β in F_{β^*} measure from a set of values:

$$\beta = \{20, 19, 18, ..., 2, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{20}\},$$

which represents that the relative weight of type I error is punished more than type II error by a range from 20 times to 1/20 times. We then search the optimal β that maximizes CER^{asy}. As shown in Figure 5, if the distribution of the unmananaged portfolio has a higher skewness, the optimal β tends to be lower.

[8]The optimal β depends on investor preference. We choose the CER^{asy} to highlight the effect of skewness. Our results are qualitatively similar but weaker when we choose performance measures ignoring return asymmetry.

5.2.2 A Stylized Model

Downside management could be appropriate for portfolios that have more negatively skewed distributions since the benefit of avoiding downside could dominate the cost of missing upside, relative to the buy-and-hold strategy. Downside management could also benefit investors who are loss averse.[9] In our setting, we provide a theoretical framework to illustrate the economics of downside-managed portfolios.

Borrowing the theoretical framework from Brunnermeier and Parker (2005), we assume a two-state world with two periods, in which investors choose portfolio weights between a risk-free asset and risky assets. A risky asset is assumed to have the following payoff distribution:

State	Objective probability	Predicted probability	Payoff return in period 2
Downside	p	π	$Z_d = \mu - \sigma\sqrt{\frac{1-p}{p}}$
Normal	$1-p$	$1-\pi$	$Z_u = \mu + \sigma\sqrt{\frac{p}{1-p}}$

In this setting, the payoff has fixed mean, μ, and variance, σ, and its skewness increases in the probability of the downside state. To isolate the effect of skewness on portfolio choice, we set $\mu = 0$. In period one, the investor adopts a model with the predicted probability of the downside state being π and chooses the optimal weight α to maximize her expected utility:

$$\max_{\alpha} \pi u(1+\alpha Z_d) + (1-\pi)u(1+\alpha Z_u), \quad s.t.\, 0 < \pi < 1,$$

the first-order condition of the maximazation problem is:

$$\pi u'(1+\alpha Z_d)Z_d + (1-\pi)u'(1+\alpha Z_u)Z_u = 0.$$

[9] On an aggregate level, many studies document that the representative investor places higher weights on losses than on gains when assessing her portfolio risk (Allais (1979); Gill and Prowse (2012)) and shows a preference for skewed assets (Kane (1982); Dahlquist et al. (2017); Mu et al. (2019)). By assuming nonstandard preference, this strand of literature provides theoretical and empirical evidence showing that investors' portfolio depends on the return distribution.

Under the first-order condition, the optimal weight allocated on the risky asset is a function of predicted probability π. The optimal predicted probability π^* is the probability that maximizes the expected time-average of utilities of the two periods:

$$\begin{aligned} \max_{\pi} \frac{1}{2}[\pi u(1+\alpha^*(\pi)Z_d) + (1-\pi)u(1+\alpha Z_u)] & \qquad \cdots \textit{anticipated EU} \\ +\frac{1}{2}[pu(1+\alpha^*(\pi)Z_d) + (1-p)u(1+\alpha Z_u)] & \qquad \cdots \textit{actual EU}, \end{aligned}$$

with the first-order condition:

$$u(1+\alpha^*(\pi)Z_u) - u(1+\alpha^* Z_d) = -[pu'(1+\alpha(\pi)Z_d)Z_d + (1-p)u(1+\alpha^* Z_d)]\frac{d\alpha^*}{d\pi}. \tag{10}$$

In our setting, the objective probability p can be estimated as a constant from the historical data. When the investor chooses downside management, π^* would deviate from p and thus the difference between the unmanaged portfolio and downside-managed portfolio.

We first demonstrate the relation between the skewness of the return distribution and investors' optimal choice and plot this relation in Figure 4. Without loss of generality, we assume $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\gamma = 3$. The left panel shows that the optimal subjective probability is negatively correlated with the skewness of the risky asset, and the right panel indicates higher skewness is associated with higher optimal weight on the risky asset.

Next, the choice of downside management depends on the investor preference, which can arise from two sources. The first source is the specification of $U(c)$, especially on the skewness of the return distribution. The second source is the relative weight between the anticipated and actual EU components, assumed to be equal in our specification, in that a higher weight on the anticipated EU component leads to stronger downside management.

In our setting, the optimal predicted probability corresponds to the choice of β. Given a model for downside prediction, a higher β corresponds to a prediction that the downside state is more likely to happen in the next period. Therefore, the skewness of the return distribution has direct implication on the optimal choice of β.

With empirical evidence and theoretical prediction, we demonstrate that investors are able to further enhance the performance of our downside-managed portfolios by ex-ante selecting a higher (lower) level of β for factors with a more (less) negatively skewed return distribution.

6 Conclusion

The volatility-managed portfolio introduced by Moreira and Muir (2017) is regarded as a groundbreaking study to demonstrate that buy-and-hold is a sub-optimal portfolio by showing the impressive performance of the volatility-timing strategy. Recent papers criticize their work in two aspects: the strategy is difficult to implement in real-time and suffers from limits to arbitrage. We propose a downside-managed portfolio constructed in a real-time scenario. Empirically, we find that our portfolios outperform the original buy-and-hold portfolios as well as volatility-managed portfolios evaluated by different performance measures. We also conduct a large set of tests following Cederburg et al. (2020) and find that our strategy is robust to different factors and anomaly portfolios. Moreover, our strategies have a higher tolerance to transaction costs and liquidity risks.

*** shall we understate everything about beta selection?*** The downside-managed portfolio is based on the optimization of the threshold to predict the downside state from the normal state by estimating downside probability. We highlighted that the optimal threshold varies on the weight of type I error relative to type II error, and the relative weight depends on the two aspects: the aversion to downside and asymmetry in return distribution. By borrowing the setting in Brunnermeier and Parker (2005), we derive a theoretical relation between the skewness of return distribution and the optimal β in F_β measure under a non-standard preference with asymmetric weight on the tails of the return distribution. In summary, we show that downside-managed portfolios offer a robust method of volatility-timing and could enable buy-and-hold investors to achieve higher utility gains.

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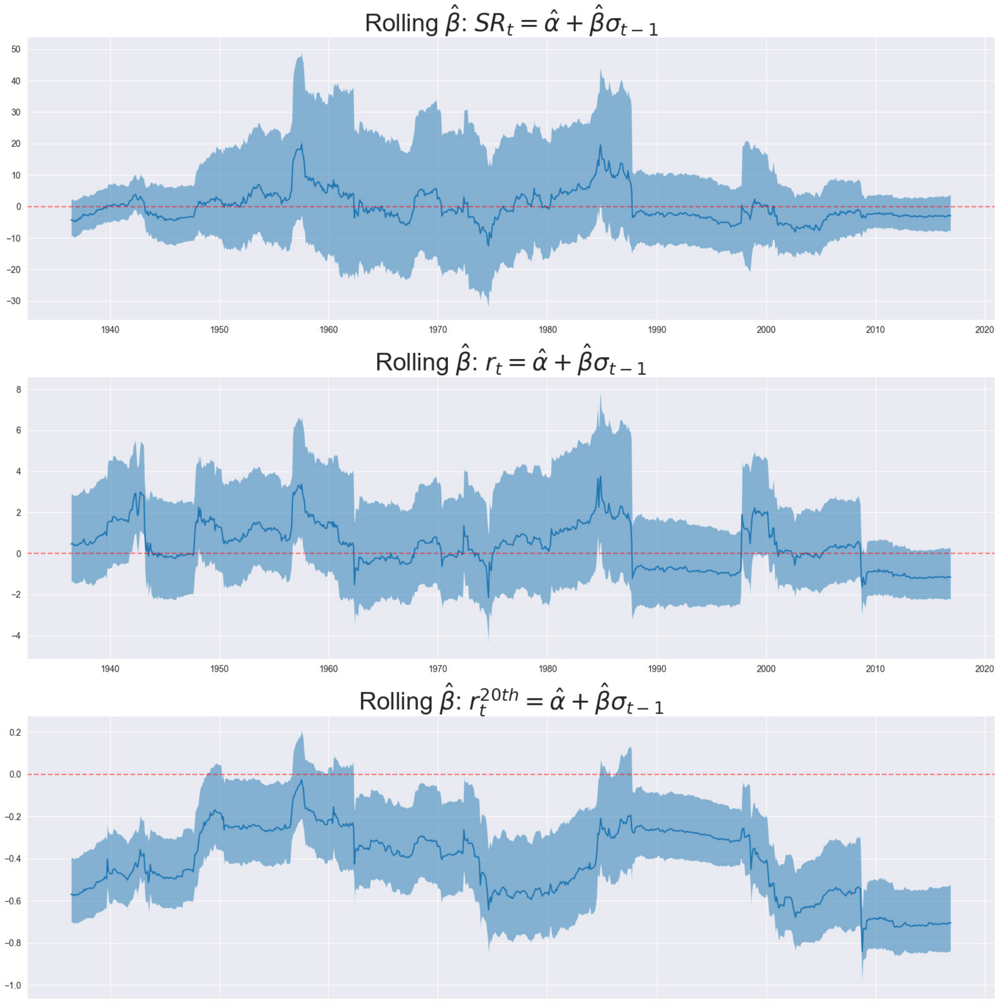

Figure 1: Rolling Covariance: MKT factor

Notes: The figure displays the estimates of coefficients of three sets of rolling regressions. The left-hand variables in the three panels represent the Sharpe ratio, SR_t, in month t, 20th percentile of daily returns, r_t^{20th}, in month t, and the total return, r_t, in month t, respectively.

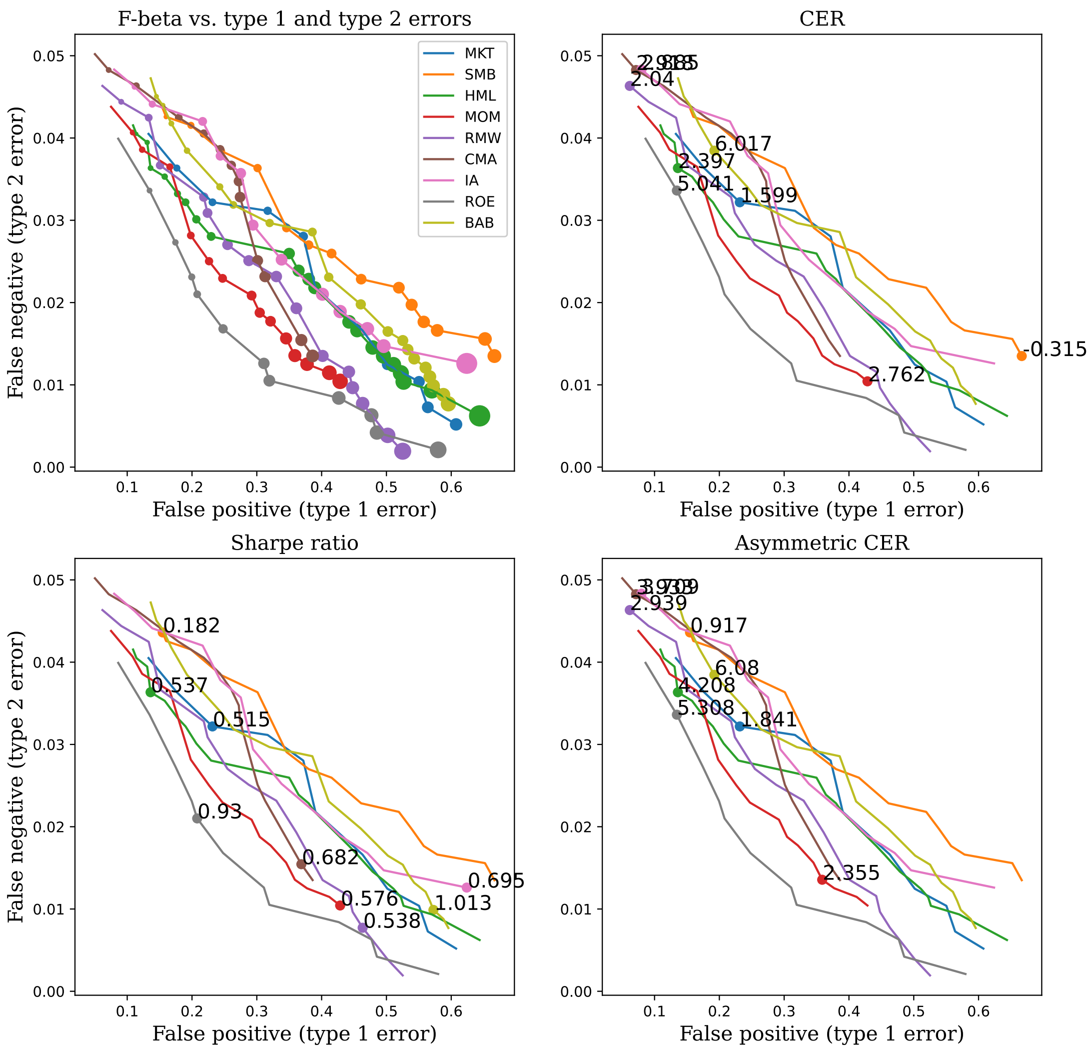

Figure 2: Model performance under different levels of beta: Nine factors

Notes: The figure shows the rate of type I and type II errors ($\frac{\#FP}{\#Total}$ and $\frac{\#FN}{\#Total}$) by maximizing F_β measure with different levels of β for each factor. In the top left panel, the size of the marker represents the value of β (larger size stands for higher β). In the remaining panels, we mark the location of optimal β choice to maximize the SR, CER, and CER^{asy}, and annotate the corresponding value of the performance metrics.

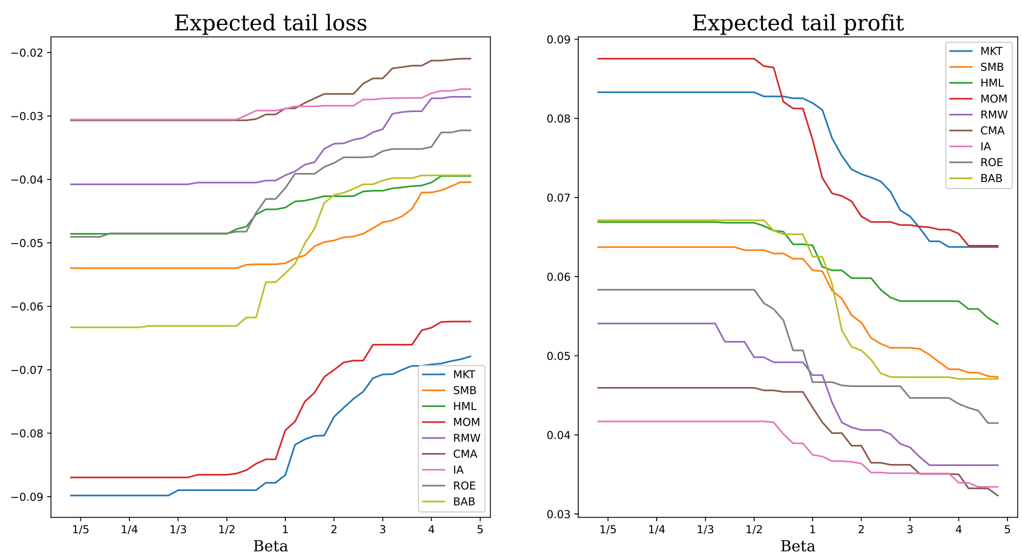

Figure 3: Tails density and F_β measure

Notes: The figure displays the tail density for nine factors. For each factor, we obtain expected tail loss (expected tail profit) by calculating the mean of all returns that are lower (higher) than the 5th (95th) percentile of the downside-managed portfolio returns that are calculated with different β in F_β.

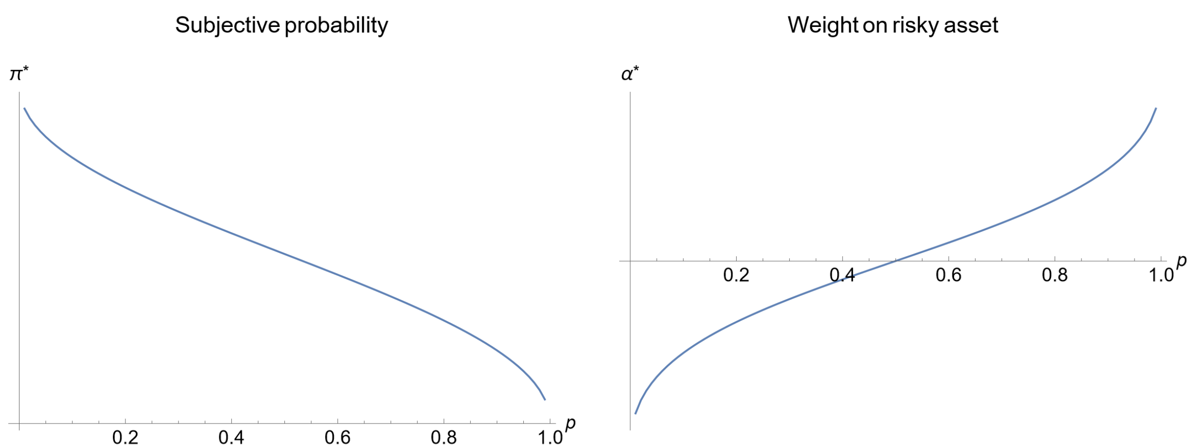

Figure 4: Relation at optimum

Notes: The figure displays the optimal subjective beliefs and optimal weight on the risky asset at different levels of skewness of the terminal payoff.

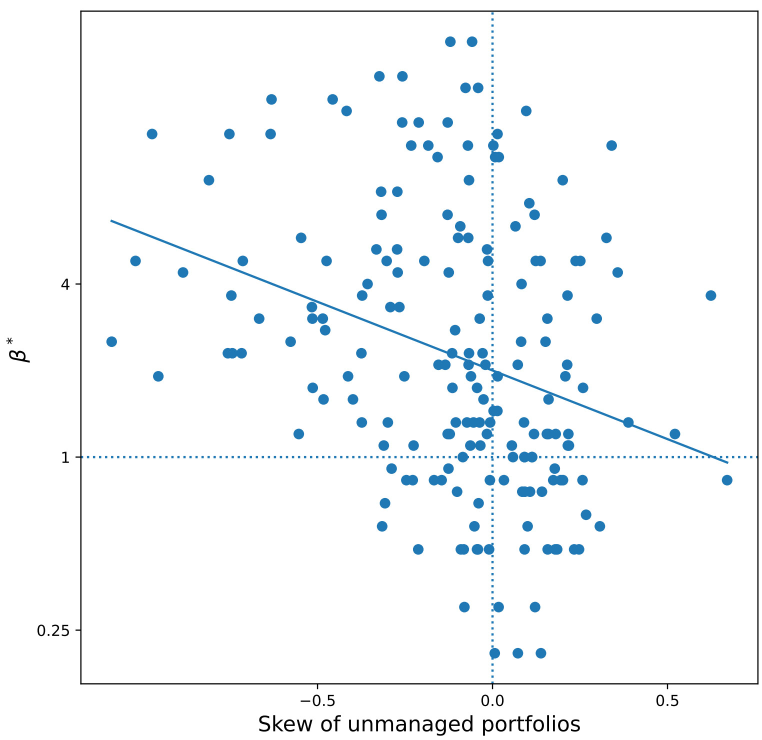

Figure 5: Skewness and Optimal β: 187 Anomaly Portfolios

Notes: The figure displays the skewness of 187 anomaly portfolios and the corresponding optimal level of β maximizing the CER^{asy}.

Table 1: Regression Coefficients: Total Return vs. VaR

The table shows the coefficient estimation of two sets of regression for nine major factors:

$$\begin{aligned} SR_t &= \hat{\alpha} + \hat{\beta}\sigma_{t-1}; \\ r_t^{total} &= \hat{\alpha} + \hat{\beta}\sigma_{t-1}; \\ r_t^{20th} &= \hat{\alpha} + \hat{\beta}\sigma_{t-1}, \end{aligned} \tag{11}$$

where SR_t is the Sharpe ratio in month t; r_t^{20th} is the 20^{th} percentile return calculated using daily returns in month t; r_t^{total} is the total return in month t. The right-hand side variable σ_t is the standard deviation of daily returns in month $t-1$. The table reports the $\hat{\beta}$ estimation, and the corresponding t-stat is reported in the bracket.

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	BAB
SR_t	-3.22	0.88	-0.58	-10.43	-5.20	4.93	2.45	-17.32	-5.72
	[-2.34]	[0.30]	[-0.22]	[-4.71]	[-0.88]	[0.73]	[0.39]	[-2.60]	[-2.18]
r_t^{total}	-0.08	0.10	0.76	-1.45	1.25	1.53	1.31	-1.01	0.18
	[-0.29]	[0.34]	[2.57]	[-4.88]	[3.00]	[3.49]	[3.06]	[-1.99]	[0.60]
r_t^{20th}	-0.58	-0.57	-0.58	-0.62	-0.60	-0.46	-0.51	-0.61	-0.61
	[-25.04]	[-27.65]	[-32.55]	[-26.54]	[-21.16]	[-16.05]	[-16.76]	[-17.71]	[-25.41]

Table 2: Real-time Performance: S&P 500 Excess Returns

The table shows the performance of the unmanaged version, downside managed versions, and volatility-managed version of the S&P500 index excess return over the risk-free rate. The table compares the performance metrics, including the mean return, standard deviation, annualized Sharpe ratio (SR), 1-year maximum drawdown (MDD), certainty equivalent returns (CER), and the certainty equivalent returns with asymmetric preferences (CER^{asy}). we follow Jobson and Korkie (1981) to calculate the p-value of the SR difference and DeMiguel et al. (2009) to compute the p-value of the CER difference. We also show the downside portfolios using the option implied predictor, calculated from (6), (7), and (8). Panel A shows the performance evaluation during the period from 1962 to 2020, while Panel B presents the period since 1996 when the option prices data period starts in the OptionMetrics dataset. The results for CER are for $\gamma = 6$, and CER^{asy} are for $\tilde{\gamma} = 7.2$ and $\tilde{\chi} = 0.0488$, according to the calibration results in Dahlquist et al. (2017)

	Mean	Std	MDD	SR	ΔSR	ΔSR p-value	CER	ΔCER	ΔCER p-value	CER^{asy}
Panel A: 1962-present										
S&P 500	0.038	0.153	-0.479	0.246			-3.232			-5.061
Downside-managed (F_1 measure)	0.039	0.114	-0.304	0.342	0.096	0.010	0.006	3.238	0.000	-1.094
Downside-managed (F_{β^*} measure)			-0.271	0.356	0.109	0.005	0.582	3.814	0.000	-0.054
Vol-managed	0.042	0.144	-0.417	0.292	0.045	0.198	-2.003	1.229	0.160	-3.998
Panel B: 1996-present										
S&P 500	0.038	0.150	-0.479	0.250						
Vol-managed	0.048	0.173	-0.417	0.278	0.028	0.226	-4.184	-1.183	0.189	-6.823
Downside-managed (F_1 measure)	0.046	0.121	-0.304	0.381	0.130	0.010	0.200	3.201	0.001	-1.200
Downside-managed (F_{β^*} measure)			-0.271	0.431	0.181	0.002	0.904	3.905	0.000	-0.399
SVIX downside-managed	0.052	0.134	-0.345	0.385	0.134	0.002	-0.237	2.763	0.000	-1.675
SVIX-Up downside-managed	0.044	0.131	-0.304	0.337	0.086	0.029	-0.736	2.265	0.004	-2.136
SVIX-Down downside-managed	0.062	0.130	-0.271	0.480	0.230	0.000	1.178	4.179	0.000	-0.041

Table 3: Real-time Performance: Nine factors

The table compares the performance metrics, including the SR, MDD, CER, and CER^{asy}. The first three panels present the performance metrics for nine original portfolios (Panel A), the baseline downside-managed portfolios (Panel B), the downside-managed portfolios with F_{β^*} measure (Panel C), and the real-time volatility-managed portfolios (Panel D). The sample period spans from 1926 to 2020. The CER results adopt for $\gamma = 6$, and CER^{asy} are for $\tilde{\gamma} = 7.2$ and $\tilde{\chi} = 0.0488$, according to the calibration results in Dahlquist et al. (2017)

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	BAB
	Panel A: Original factors								
Sharpe ratio	0.483	0.163	0.492	0.488	0.460	0.629	0.658	0.834	0.844
MDD	-0.471	-0.310	-0.313	-0.551	-0.315	-0.139	-0.137	-0.264	-0.395
CER	0.194	-1.384	1.951	0.940	1.756	2.900	3.013	5.034	5.520
CER^{asy}	-1.745	-0.395	3.001	-1.081	2.461	3.627	3.611	4.283	4.345
	Panel B: Downside managed portfolios (F_1 measure)								
Sharpe ratio	0.511	0.173	0.521	0.530	0.496	0.663	0.661	0.899	0.946
MDD	-0.358	-0.241	-0.207	-0.378	-0.200	-0.139	-0.121	-0.201	-0.350
CER	1.333	-0.859	2.263	1.931	1.967	2.930	2.754	4.863	6.017
CER^{asy}	1.452	0.777	3.963	1.741	2.824	3.914	3.474	4.777	6.002
	Panel C: Downside managed portfolios (F_{β^*} measure)								
Sharpe ratio	0.517	0.186	0.537	0.584	0.553	0.700	0.702	0.947	1.019
MDD	-0.358	-0.238	-0.194	-0.328	-0.168	-0.087	-0.094	-0.139	-0.217
CER	1.615	-0.258	2.397	2.844	2.252	3.009	3.000	5.171	6.017
CER^{asy}	1.669	0.871	4.134	2.350	3.501	3.996	3.792	5.253	6.002
	Panel D: Volatility-managed portfolios								
Sharpe ratio	0.303	0.165	0.332	0.539	0.556	0.474	0.620	1.024	0.900
MDD	-1.375	-0.391	-0.532	-0.507	-0.243	-0.303	-0.240	-0.131	-0.486
CER	-9.429	-2.322	-1.349	0.386	2.454	1.872	2.967	6.949	6.569
CER^{asy}	-16.143	-0.428	1.346	-2.388	3.170	2.573	3.825	8.594	7.442

Table 4: Performance Comparison: Nine Factors

The table shows the pairwise performance comparison of the SR, CER, and corresponding p-value of difference. The calculation of p-value of Sharpe ratio difference follows Jobson and Korkie (1981) approach, and the calculation of p-value of CER follows DeMiguel et al. (2009) approach. and CER^{asy} are for $\tilde{\gamma} = 7.2$ and $\tilde{\chi} = 0.0488$, according to the calibration results in Dahlquist et al. (2017)

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	BAB
Panel A: Sharpe ratio comparison									
Downside vs. Orginal									
Dif	0.028	0.010	0.030	0.042	0.036	0.034	0.003	0.066	0.102
p-value	0.065	0.178	0.062	0.016	0.063	0.022	0.228	0.002	0.000
Vol-managed vs. Orginal									
Dif	-0.180	0.002	-0.160	0.051	0.097	-0.155	-0.038	0.191	0.056
p-value	0.014	0.245	0.021	0.156	0.086	0.027	0.173	0.018	0.145
Downside vs. Vol-managed									
Dif	0.208	0.008	0.190	-0.009	-0.061	0.189	0.041	-0.125	0.045
p-value	0.006	0.231	0.007	0.232	0.123	0.008	0.161	0.045	0.158
Panel B: CER comparison									
Downside vs. Orginal									
Dif	1.139	0.525	0.312	0.991	0.211	0.029	-0.259	-0.171	0.498
p-value	0.006	0.019	0.077	0.005	0.153	0.229	0.102	0.184	0.043
Vol-managed vs. Orginal									
Dif	-9.624	-0.938	-3.299	-0.554	0.698	-1.028	-0.046	1.915	1.049
p-value	0.000	0.084	0.002	0.187	0.130	0.073	0.241	0.042	0.099
Downside vs. Vol-managed									
Dif	10.762	1.463	3.612	1.545	-0.487	1.057	-0.213	-2.085	-0.551
p-value	0.000	0.032	0.000	0.083	0.142	0.057	0.203	0.018	0.157

Table 5: Performance Comparison: Broad Sample

The table shows pairwise performance comparisons of the SR, MDD, CER, and CER^{asy} among 94 original portfolios, the corresponding downside-managed portfolios, and the corresponding real-time volatility-managed portfolios. For each set of comparisons, the table presents the total count of performance metrics differences that are positive and negative. The number in each parenthesis represents the count of significant positive or significant negative differences at the 5% level. The statistical significance of Sharpe ratio differences and CER differences are from Jobson and Korkie (1981) approach and DeMiguel et al. (2009) approach. and CER^{asy} are for $\tilde{\gamma} = 7.2$ and $\tilde{\chi} = 0.0488$, according to the calibration results in Dahlquist et al. (2017)

		Downside vs. original		Downside vs. vol-managed		Vol-managed vs. Original	
	total	SR dif >0	SR dif <0	SR dif >0	SR dif <0	SR dif >0	SR dif <0
Panel A: Sharpe ratio comparison							
Accruals	10	6[3]	4[1]	7[4]	3[0]	3[0]	7[4]
Intangibles	10	8[4]	2[0]	8[5]	2[0]	2[2]	8[4]
Investment	9	5[2]	4[1]	7[5]	2[1]	2[1]	7[4]
Momentum	8	8[7]	0[0]	3[1]	5[1]	6[5]	2[0]
Profitability	20	15[6]	5[1]	10[5]	10[3]	14[3]	6[5]
Trading	19	12[7]	7[4]	11[2]	8[3]	8[3]	11[3]
value	18	12[3]	6[1]	11[3]	7[1]	10[1]	8[2]
All trading strategies	94	66[32]	28[8]	57[25]	37[9]	45[15]	49[22]
Panel B: CER comparison							
Accruals	10	7[6]	3[0]	10[7]	0[0]	3[0]	7[6]
Intangibles	10	10[7]	0[0]	9[8]	1[0]	2[0]	8[5]
Investment	9	8[3]	1[0]	7[7]	2[0]	2[0]	7[2]
Momentum	8	8[7]	0[0]	4[3]	4[0]	5[3]	3[2]
Profitability	20	18[13]	2[0]	16[6]	4[2]	13[2]	7[3]
Trading	19	19[16]	0[0]	17[15]	2[0]	4[2]	15[9]
Value	18	18[14]	0[0]	17[8]	1[0]	11[1]	7[3]
All trading strategies	94	88[66]	6[0]	80[54]	14[2]	40[8]	54[30]
Panel C: CER^{asy} comparison							
Accruals	10	6	4	7	3	2	8
Intangibles	10	10	0	8	2	2	8
Investment	9	7	2	7	2	2	7
Momentum	8	8	0	2	6	7	1
Profitability	20	18	2	12	8	14	6
Trading	19	16	3	16	3	5	14
Value	18	16	2	15	3	10	8
All	94	81	13	67	27	42	52

Table 6: Transaction costs: S&P 500 index

The table reports the Sharpe ratio of downside-managed portfolios and volatility-manged portfolios after accounting for transaction costs. We consider 5 levels of transaction costs: 1 bps, 10 bps, 14 bps, 25 bps, and 50 bps. All results are in annualized terms.

	Vol-managed	Downside	Downside SVIX	Downside SVIX-Up	Downside SVIX-Down
			1962-present		
SR_{1bps}	0.288	0.341			
SR_{10bps}	0.256	0.334			
SR_{14bps}	0.241	0.331			
SR_{25bps}	0.201	0.323			
SR_{50bps}	0.110	0.303			
			1996-present		
SR_{1bps}	0.274	0.380	0.385	0.336	0.480
SR_{10bps}	0.242	0.376	0.382	0.333	0.478
SR_{14bps}	0.227	0.374	0.381	0.331	0.477
SR_{25bps}	0.188	0.369	0.379	0.327	0.474
SR_{50bps}	0.097	0.358	0.372	0.317	0.467

Table 7: Turnover ratios and break-even transaction costs: S&P 500 index

The table reports the implied transaction costs needed to drive each performance metric (SR, CER, and CER^{asy}) to zeros. The table also reports the average absolute change in monthly weights of the corresponding portfolios.

	Vol-managed	Downside	Downside SVIX	Downside SVIX-Up	Downside SVIX-Down
		1962-present			
Turnover	0.437	0.073			
$SR_{breakeven}$	12.67	123.76			
$CER_{breakeven}$	21.67	299.78			
$asyCER_{breakeven}$	20.27	410.92			
		1996-present			
Turnover	0.525	0.044	0.027	0.042	0.031
$SR_{breakeven}$	7.82	291.31	538.32	220.85	807.62
$CER_{breakeven}$	-12.64	486.11	695.90	374.28	1009.00
$asyCER_{breakeven}$	-31.94	568.84	805.70	475.79	1099.00

Table 8: Turnover ratios and break-even transaction costs: Nine major factors

The table reports the implied transaction costs needed to drive each performance metric (SR, CER, and CER^{asy}) to zeros. The table also reports the average absolute change in monthly weights of the corresponding portfolios.

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	BAB
	Panel A: Downside portfolios								
Turnover	0.056	0.070	0.059	0.062	0.053	0.042	0.067	0.078	0.067
$SR_{breakeven}$	58.31	11.04	36.45	70.91	37.64	41.16	2.49	45.38	109.98
$CER_{breakeven}$	167.93	63.04	45.30	129.59	33.37	6.28	-32.60	-17.12	61.74
$asyCER_{breakeven}$	447.43	139.97	136.88	362.76	63.05	50.76	-15.31	48.34	203.32
	Panel B: Vol-managed portfolios								
Turnover	0.647	0.512	0.65	0.816	0.512	0.59	0.583	0.578	0.602
$SR_{breakeven}$	-55.37	0.44	-29.90	9.11	11.51	-16.84	-4.23	28.82	10.19
$CER_{breakeven}$	-133.31	-15.71	-44.44	-5.68	11.51	-14.50	-0.62	29.77	15.05
$asyCER_{breakeven}$	-202.37	-0.52	-21.46	-10.69	10.99	-14.58	3.12	62.33	27.82

Table 9: Break-even transaction costs comparison: 187 portfolios downside managed vs. vol-managed

The table reports the comparison of break-even transaction costs between downside-managed portfolios and volatility-manged portfolios. The comparison is applied to 187 portfolios. The number in the table represents the total count of break-even transaction cost differences that are positive and negative.

		Tunover ratio		SR break-even		CER break-even		CER^{asy} break-even	
	Total	dif >0	dif <0	dif >0	dif <0	dif >0	dif <0	dif >0	dif <0
Accruals	10	0	10	8	2	10	0	7	3
Intangibles	10	0	10	8	2	10	0	10	0
Investment	9	0	9	6	3	9	0	6	3
Momentum	8	0	8	8	0	8	0	7	1
Profitability	20	0	20	15	5	19	1	19	1
Trading	19	0	19	12	7	19	0	18	1
Value	18	0	18	14	4	18	0	16	2
All	94	0	94	71	23	93	1	83	11